Special Feature A
Sectoral, Industry and Employment Business Cycles in Singapore

Introduction

The impact of the current global financial crisis is spreading rapidly across many industries in the domestic economy, while the labour market has also shown signs of deterioration recently. This special feature carries out an in-depth study of Singapore’s sectoral, industry and employment business cycles in order to shed some light on the following questions. How quickly will external demand shocks diffuse through the Singapore economy? Which sectors are likely to be the most severely hit? Will all industries suffer from retrenchments and rising unemployment or are some better insulated?

Most studies on Singapore’s business cycle have focused on the correlations between domestic and external aggregate fluctuations (see, for example, Choy, 2009). There is no known research which deals specifically with inter-sectoral co-movements in output and employment, despite these being an integral aspect of business cycles.²

This special feature applies a frequency domain band-pass filter to isolate cyclical components in foreign economic indicators, as well as domestic sectoral and industry data. The cycles which are extracted are then analysed through bivariate and multivariate correlation statistics.

This approach differs from most methodologies employed in studies of economic fluctuations, which use time domain-based filters, such as the Hodrick-Prescott filter, to identify cyclical movements in macroeconomic data.

An investigation of the boom-bust cycles in production and employment is important for several reasons. First, it facilitates a careful assessment of the degree of synchronization of world and regional business cycles with Singapore’s macroeconomic fluctuations. If sector-wide cycles correlate most strongly with worldwide gyrations, for example, it must be concluded that growth is driven ultimately by global forces rather than regional or domestic impulses.

Second, a disaggregated analysis of sectoral business cycles can help to improve forecasts. In particular, it would be easier to forecast sectors that exhibit high correlations with real GDP by anticipating the broad trends in economic growth.

Third, an analysis of the amplitudes and volatilities of employment cycles in the major sectors can help identify those industries in which employment is most vulnerable to economic downturns and those that offer more stable job prospects. It will also enable us to quantify the spillover effects on job creation as a result of inter-dependency between industries, strategic complementarities or simply exposure to common shocks.

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1 This Special Feature was done in collaboration with Assistant Professor Choy Keen Meng from the Division of Economics, Nanyang Technological University. Professor Sam Ouliaris from the National University of Singapore also provided useful comments.

2 Hornstein (2000) and Christiano and Fitzgerald (1998) have carefully documented the cyclical behaviour of such variables for the US economy.
The Frequency Domain Filter

To study the co-movements amongst sectoral aggregates, associated business cycles need to be extracted in a statistically rigorous manner. This is done by using the frequency domain (FD) filter developed by Corbae and Ouliaris (2006). Like all ideal band-pass filters, the FD technique isolates the cyclical component in time series by retaining fluctuations of a specified duration and eliminating the rest. Following the literature, only economic cycles with duration of 6 to 32 quarters are used in this feature.\(^3\)

The FD filter has the advantage over other time-domain based methods, such as the Hodrick-Prescott (HP) filter, in that it preserves data points at both ends of the time series.\(^4\) This is essential for real-time business cycle monitoring and analysis without introducing phase shift into the observations (see Box D). More importantly, the FD filter was designed to extract cycles from non-stationary series with stochastic (or deterministic) trends, which makes it ideal for the Singapore data set.\(^5\)

The dynamic relationships between the estimated business cycles can then be described using time series plots and, more formally, by computing bivariate cross correlation coefficients at various leads and lags between sectoral, industry and employment variables. Furthermore, historical standard deviations of the business cycle components in different sectors will indicate how cyclically volatile each sector is.

However, at times, it is insufficient to examine the cross correlations for a pair of economic aggregates. For example, it might be useful to investigate the co-movements between a set of global demand variables on the one hand, and Singapore’s services-producing industries on the other. In such instances, it is more instructive to calculate the multivariate canonical correlations between the two groups of variables. This is achieved by isolating linear functions of the variables in one group that maximally correlate with linear functions in the other group, otherwise known as canonical variates. The canonical correlations are simply the sample correlation coefficients between the variates obtained and provide information on the strength of the interrelations between the economic variables in the two data sets.\(^6\) In the following sections, both cross and canonical correlations are reported.

Data

Seasonally adjusted quarterly data spanning 1975 to 2008, or sub-periods thereof, are analysed in this special feature. These are obtained from published time series and internal estimates compiled by EPD. All data are expressed as logarithms.

The EPD’s composite foreign GDP index is used to capture the world business cycle, a regional GDP sub-index to represent the state of the Asian economies, and global chip sales as a proxy for the world electronics cycle. Singapore’s real GDP is taken to be the most comprehensive measure of

\(^3\) Choy (2009) found that business cycles with a specified duration of 6 to 32 quarters are applicable to Singapore.

\(^4\) In comparison, the HP filter potentially distorts the timing of the turning points in a business cycle due to its asymmetric nature.

\(^5\) Standard tests do not reject the null hypothesis that a unit root is present in all the economic time series analysed.

\(^6\) See Manly (2005) for an accessible introduction to the method of canonical correlations.
economic activity. The goods-producing sectors are manufacturing and construction, which account for 25% and 4% of total output in real terms, respectively. Within manufacturing, the key industry clusters are: electronics, chemicals, biomedical products, precision engineering, and transport engineering.

The services-producing sector is disaggregated into commerce, transport & communications, and financial and business services, contributing 18%, 14% and 25% to GDP, respectively. Where appropriate, these are further divided into wholesale & retail trade, hotels & restaurants, transport & storage, information & communications, financial intermediation, and business services.

In the canonical correlation analysis, activity indicators, such as tourist arrivals and retail sales volumes, are also used to supplement the proxies for foreign and domestic demand. Finally, to investigate co-movements in the labour market, employment statistics on key sectors of the economy are filtered and their correlations examined.

**Empirical Findings**

Chart 1 compares the business cycle components in Singapore’s real GDP that are extracted by the FD and standard HP filters. The cyclical component obtained from the FD filter is a smooth curve that alternates between the peaks and troughs of economic activity. In comparison, the cyclical component extracted by the HP filter is highly irregular, evidenced by the choppy curve. Nonetheless, the two cycles have similar turning points and they both indicate that the Singapore economy has become more volatile in recent years. Their pairwise correlation is 0.92.

Chart 1 also highlights the cycles in Singapore GDP including the downturns associated with the oil shocks of the 1970s, the mid-eighties recession, the 1990-92 Gulf war, the Asian Financial Crisis of 1997-98, the IT collapse of 2000-01, the SARS epidemic in 2003, and the current global financial crisis. Since the data is in logarithms, the amplitudes of the cycles represent the percentage changes in GDP. According to this measure, the most severe recession was in the mid-1980s, when output growth fell by nearly 12% points from peak to trough, although this is likely to be surpassed by the current recession.
The World Economy and Singapore

The perception that external events have a crucial influence on Singapore’s business cycle is confirmed in Chart 2. The chart at the top left suggests that the cyclical component of Singapore’s GDP is essentially driven by world business cycles and global electronic cycles, with their turning points and even minor fluctuations mapping well onto each other.

This hypothesis is supported by the estimated contemporaneous cross correlation between world GDP and global chip sales on the one hand, and Singapore’s GDP on the other. At 0.60, this coefficient is the largest amongst the cross correlations, suggesting that pure external demand shocks and electronics disturbances are rapidly transmitted to the domestic economy within three months. However, fluctuations in chip sales are far more volatile (standard deviation of 15%) than fluctuations in world GDP (1.1%) or domestic GDP (2.8%).

The chart at the top right shows that both regional GDP and ASEAN-4 GDP have cross correlations with Singapore’s GDP fluctuations of less than 0.50. It therefore appears that Singapore is more dependent on the world economy and the global semiconductor market than on the region per se.

The second row in Chart 2 plots world GDP vis-à-vis Singapore’s manufacturing and services value-added. Clearly, the manufacturing sector tracks foreign economic cycles better than the services sector does, with a cross correlation of 0.65 in the former compared to 0.53 in the latter. This confirms the stylized fact that goods-producing industries are more cyclically-sensitive than services. Manufacturing also tends to be more volatile, as borne out by its standard deviation of 6.1%, compared to only 2.0% for services.

Similar comments can be made about the co-movements of these two sectors with respect to regional GDP and world semiconductor sales. For example, the contemporaneous cross correlation coefficient between domestic manufacturing output and global chip sales is high at 0.74. Moreover, business cycles in the domestic manufacturing sector appear more synchronised with Asian economic growth than those in the services sector. This may reflect Singapore’s key role in the regional electronics supply chain.

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7 Estimated cross correlation coefficients in this special feature that exceed 0.20 are statistically significant at the 5% level.

8 Regional GDP comprises NIEs (Hong Kong, Korea and Taiwan), ASEAN-4 (Indonesia, Malaysia, Philippines and Thailand), China and India.
Chart 2
World versus Singapore Business Cycles

World GDP, Global Chip Sales and Singapore GDP

Regional, ASEAN-4 and Singapore GDP

World GDP and Singapore Manufacturing GDP

World GDP and Singapore Services GDP

Regional GDP and Singapore Manufacturing GDP

Regional GDP and Singapore Services GDP

Global Chip Sales and Singapore Manufacturing GDP

Global Chip Sales and Singapore Services GDP

Note: Figures in parentheses show the maximum cross correlations and lags between the two series in the chart. Those without lags indicate a contemporaneous relationship.
**Sectoral and Industry Co-movements**

This section documents the cyclical behavior of the major sectors in the Singapore economy. In Chart 3, the FD filtered cycles for key sectors of the economy are compared against those obtained from actual domestic GDP. It is clear that all sectors, apart from construction, exhibited abrupt declines by the end of 2008, consistent with the NBER definition of an economic recession as being “marked by widespread contractions in many sectors of the economy”.

The first graph shows that the manufacturing sector has taken the brunt of the downturn thus far. The simple correlation between industrial production and real GDP is 0.79. In comparison, the maximum cross correlation between construction output and aggregate GDP is only 0.53 and this comes after a one-year lag, owing to the long lead-time for construction investment. In fact, the certified payments used to estimate construction value-added are typically made 3-4 quarters after contracts are awarded, explaining the delayed pattern in peaks and troughs in Chart 3.

Of the other major sectors considered, commerce has the highest correlation with aggregate output (0.87) and also the highest standard deviation (3.8%). It may be inferred that wholesale and retail trade, which makes up 90% of this sector’s value-added, is sensitive to overseas demand as well as the global electronics cycle, the latter because of the high proportion of semiconductor intermediate parts and components in Singapore’s re-export trade. A positive co-movement with overall GDP is also observed for transport and storage.

The financial and business services sector appears to follow real GDP relatively closely with a correlation coefficient of 0.51 and at a one-quarter lag. However, it turns out that the financial services’ filtered correlation with total output is only 0.30 and it has a high volatility of 5.2%. Given Singapore’s status as a major financial centre and its high degree of capital mobility, this suggests that the financial sector is subject to shocks that are exogenous to domestic GDP, such as disturbances in international financial markets. The co-movement of financial and business services with GDP thus results from the more domestically-oriented business services sub-sector such as real estate, professional, and IT services.
Chart 3
Domestic GDP versus Sectoral Business Cycles

Manufacturing

Construction

Commerce

Wholesale & Retail Trade and Hotels & Restaurants

Transport & Communications

Info & Communications and Transport & Storage

Financial & Business Services

Financial and Business Services

Note: Figures in parentheses are the maximum cross correlations and lags between the two series in the chart.
* Indicates the maximum cross correlations when construction, financial & business services and business services sectors each lag overall GDP by k=n lags.
In summary, Singapore’s sectoral production patterns are mostly procyclical and tend to move with the business cycle. However, this does not explain the underlying drivers of the industry-specific cycles in each sector. For example, output in many domestic enterprises is closely tied to the global IT sector, since the electronics industry accounts for a sizeable proportion of GDP and has significant linkages with the rest of the economy. In particular, the precision engineering cluster in manufacturing, entrepôt trade and transport hub services are especially vulnerable to fluctuations in world semiconductor demand.

A second group of service industries is also strongly orientated towards the external economic environment in general and hence is expected to be significantly correlated with variables measuring foreign income and international tourist traffic. It includes financial advisory and wealth management services, and offshore banking. The tourism-related industries include air transportation, hotels and restaurants, and retail outlets.

Finally, there is an “inner core” of the economy catering to local demand which typically benefits from buoyant conditions in the local labour, property and stock markets. It consists mainly of domestic banking and insurance activities, the general retail business, business services, and property and construction.

The drivers of sectoral business cycles are summarised in Figure 1. The classification into IT versus non-IT based industries, externally-oriented services, and domestic-oriented industries, has been adopted in recent issues of the Review to analyse the evolution of domestic economic developments. The objective here is to cross validate this categorisation through a canonical correlation analysis. Accordingly, Table 1 presents the canonical coefficients between sectoral value-added series and composite indicators of the electronics cycle (global chip sales and the US Tech Pulse Index), global demand (foreign GDP and visitor arrivals), and domestic demand (Singapore GDP and retail sales). The upper triangle in each cell displays the maximum correlation coefficient for any two variable groups whilst the lower triangle contains the second strongest correlation.

From Table 1, the IT-based industries are indeed highly correlated with external demand, and in particular, electronics demand. The non-IT manufacturing clusters co-vary more with foreign demand in general. Externally-oriented services are as strongly driven by domestic demand as by global demand.

The last group of domestically-orientated industries displays large and significant canonical correlations with proxies for domestic demand as well as other demand drivers. This suggests that in a small and open economy like Singapore, even sectors that primarily serve the domestic population are not immune to foreign economic shocks.

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9 The US Tech Pulse Index, compiled by the Federal Reserve Bank of San Francisco, tracks the health of the US technology sector in a timely manner. It is computed monthly from a number of data series that move with the technology sector as a whole: investment, consumption, employment, industrial production, and shipments in the IT sector.

10 Since there are two variables in each demand category, two canonical correlation coefficients can be calculated in every case. Their statistical significance, as determined using Wilks’ lambda test, is indicated by an asterisk.

11 This could be an artefact of the level of disaggregation applied to the relevant sectors. For instance, only air transport should be included in this grouping, but we used the time series for the overall transport sector due to data unavailability.
Figure 1
Drivers of Sectoral Business Cycles

Table 1
Canonical Correlations between Pairs of Economic Variables

<table>
<thead>
<tr>
<th>Drivers</th>
<th>Sectors</th>
<th>IT Manufacturing and Services</th>
<th>Non-IT Manufacturing</th>
<th>External Oriented Services</th>
<th>Domestic Oriented Industries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electronics Demand</td>
<td>0.71*</td>
<td>0.18</td>
<td>0.50*</td>
<td>0.78*</td>
<td>0.83*</td>
</tr>
<tr>
<td>External Demand</td>
<td>0.69*</td>
<td>0.21</td>
<td>0.77*</td>
<td>0.86*</td>
<td>0.84*</td>
</tr>
<tr>
<td>Domestic Demand</td>
<td>0.34*</td>
<td>0.15</td>
<td>0.54*</td>
<td>0.94*</td>
<td>0.91*</td>
</tr>
</tbody>
</table>

Note: The number at the upper right of each cell is the first correlation coefficient while the number at the lower left is the second coefficient.
* Indicates significance at the 5% level.
**Employment Cycles**

A further understanding of the dynamic economic relationships between industries can be obtained from a cross-sectoral analysis of employment.

Chart 4 shows that the FD filtered sectoral employment growth cycles in Singapore, excluding construction, tend to track one another fairly well.

In contrast, the employment cycles in construction appear to be counter-cyclical in relation to other sectors. These fluctuations have also been large, since the sector has gone through severe boom and bust cycles in the last three decades. This is corroborated by the standard deviation statistics in Table 2a. However, the relatively high ratio of foreign to resident workers in this sector suggests that migrant workers tend to act as an effective buffer during a recession.

At the other extreme, the wholesale and retail trade sub-sector, which employs around 15% of the Singapore workforce, provides the most stable job prospects in spite of its high output volatility. The hotels and restaurants sub-sector also has relatively low employment volatility, although it can be susceptible to unpredictable swings in employment as a result of sudden drops in tourist arrivals during crises, such as the Gulf War and the SARS epidemic.

Next, the employment cycles are compared across the sectors to assess the presence of spillover effects. The positive and generally large cross correlation coefficients in Table 3 provide empirical evidence that employment across the sectors expand and contract simultaneously in tandem with the ebb and flow of their respective business cycles. The current recession illustrates this synchronised labour market response very starkly: excluding construction, every sector experienced declines in employment during the second half of 2008. (Chart 4)

The only lagged relationship for the employment aggregates is between wholesale and retail trade and business services. The maximum correlation at the one-quarter lead is 0.66, implying that business services is stimulated by increased trade activity. Table 2b shows that the business services sector has been a stable and important source of job creation, with employment growing at an average of 7.5% p.a. over 1991-2008. The employment growth in business services has also been underpinned by job creation in the manufacturing and financial services industries, as suggested by their large pairwise correlations of 0.93 and 0.88, respectively. This in all likelihood reflects the “servicisation” of manufacturing activities, which have generated new demand for professional consultancy and advertising services. Financial services has also created the need for supporting business services such as legal and accounting.

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12 Construction is excluded because of its counter-cyclicality. Transport & communications is excluded due to data constraints.
Chart 4
Sectoral Employment Cycles

Table 2a
Volatility of Sectoral Employment Cycles, 1991 - 2008

<table>
<thead>
<tr>
<th>Sectoral Employment</th>
<th>Standard Deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Business Services</td>
<td>4.1</td>
</tr>
<tr>
<td>Construction</td>
<td>8.8</td>
</tr>
<tr>
<td>Financial Services</td>
<td>4.7</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>5.1</td>
</tr>
<tr>
<td>Wholesale &amp; Retail Trade</td>
<td>2.5</td>
</tr>
<tr>
<td>Hotels &amp; Restaurants</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Table 2b

<table>
<thead>
<tr>
<th>Sectoral Employment</th>
<th>Average Employment Growth (%)</th>
<th>Average Share of Total Employment (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Business Services</td>
<td>7.5</td>
<td>10.0</td>
</tr>
<tr>
<td>Construction</td>
<td>6.8</td>
<td>12.0</td>
</tr>
<tr>
<td>Financial Services</td>
<td>5.9</td>
<td>4.6</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>1.8</td>
<td>22.0</td>
</tr>
<tr>
<td>Wholesale &amp; Retail Trade</td>
<td>2.6</td>
<td>15.4</td>
</tr>
<tr>
<td>Hotels &amp; Restaurants</td>
<td>3.5</td>
<td>5.9</td>
</tr>
</tbody>
</table>

Table 3
Maximum Contemporaneous Cross Correlation Coefficients between Sectoral Employment Growth Cycles

<table>
<thead>
<tr>
<th></th>
<th>Business Services</th>
<th>Financial Services</th>
<th>Manufacturing</th>
<th>Wholesale and Retail Trade</th>
<th>Hotels and Restaurants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Business Services</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial Services</td>
<td>0.88</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.93</td>
<td>0.95</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wholesale and Retail Trade*</td>
<td>0.66 (k=+1)</td>
<td>0.87</td>
<td>0.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hotels and Restaurants</td>
<td>0.84</td>
<td>0.94</td>
<td>0.91</td>
<td>0.75</td>
<td></td>
</tr>
</tbody>
</table>

Note: Estimated cross correlation coefficients that exceed 0.20 are statistically significant at the 5% level.
*The coefficient for the wholesale and retail trade versus business services is at the lead of one quarter.
Sum-up

This special feature has shown the importance of using sectoral data to extract additional information about macroeconomic fluctuations in Singapore. The results corroborate the key findings of Choy (2009) that domestic business cycles are, by and large, imported from the rest of the world. Regional business cycles by themselves do not appear to have a strong influence on Singapore’s domestic business cycle, suggesting that there could be further scope for expansion of services exports to Asia.

Without attempting to identify the sources of Singapore’s business cycles – whether they result from aggregate shocks or are industry and sector-specific – there is evidence of strong co-movements in sectoral value-added (or commodity outputs) and employment (or factor inputs) across the major sectors of the economy. That said, the sectoral fluctuations of some clusters of industries are more intrinsically linked together than others. Specifically, the classification of sectors into IT-based industries, non-IT based manufacturing, externally-oriented services and domestic-oriented sectors, is broadly validated by a canonical correlation analysis. However, there is no sector or industry that is completely sheltered from foreign economic influences.

Finally, the most volatile sectors in terms of employment turnover are construction, manufacturing and financial services. While swings in construction are more innate to the industry itself and the impact on the resident workforce is cushioned to some degree by the presence of foreign workers, the relatively large fluctuations in other sectors of the economy suggest that a strategy of diversifying the scope of activities and their markets, including a renewed focus on the region as a market, may help to mitigate the ups and downs of employment cycles in Singapore.

Box D
Filtering in the Frequency Domain

Most methods of analysing economic time series operate in the time domain, in which the behavior of a series is described in terms of the way in which observations are related statistically over time. The method used in this special feature to extract business cycles works in the frequency domain instead, in that it seeks to describe the fluctuations in the series in terms of sinusoidal behavior at various frequencies. Essentially, frequency domain techniques decompose a time series into periodic components, whether or not the data appears periodic.

Frequency domain filtering has advantages and disadvantages. On the plus side, no approximations are involved, no observations are lost at the end-points of series, and it does not require the researcher to set any parameters. But on the negative side, the filtered series depends on the sample size. Hence, when new information arrives, the cyclical component needs to be redefined. Baxter and King (1999) considered this to be a drawback of frequency domain filters because it violates their requirement that the ideal band-pass filter should yield components unrelated to the length of the sample.

The other complication of the frequency domain method is that one has to decide whether to remove a stochastic and/or deterministic trend prior to filtering. Detrending is made necessary by the fact that the spectrum of a non-stationary series is undefined at the zero frequency and consequently, the discrete Fourier transform of the series contains a bias that is due to the unit root at the same frequency. The key asymptotic result that expresses this bias is the following lemma from Corbae, Ouliaris and Phillips (2002):

$$w,(l) = \frac{1}{1 - e^{i\lambda}} w,(l) - \frac{e^{i\lambda} (\bar{x} - \bar{y})}{1 - e^{i\lambda} \frac{\sqrt{n}}{\sqrt{2}}} \sqrt{n}$$
where \( w_i(\lambda_i) \) is the discrete Fourier transform of an I(1) series \( \tilde{x}_i \) and \( \lambda_i = \frac{2\pi i}{n}(s = 1,...,n-1) \) are the fundamental frequencies for sample size \( n \). The equation shows that the frequency response function of an integrated process are not asymptotically independent across frequencies by virtue of the component \( \tilde{x}_i / \sqrt{n} \). Consequently, frequency domain estimates of the business cycle will be inconsistent. Moreover, Corbae, Ouliaris and Phillips (2002) prove that this bias does not disappear even if the original series is first detrended in the time domain.

Corbae and Ouliaris’ (2006) solution to this problem is to detrend the second term in the above expression via a frequency domain regression, thereby yielding an unbiased estimate of \( w_i(\lambda_i) \). They also show that this estimate is \( \sqrt{n} \)-consistent and demonstrated through Monte Carlo simulations that it has good finite sample properties. An indicator function can then be applied to the estimate to cut out the fluctuations in time series that do not correspond to the definition of business cycles as oscillations with durations of 6-32 quarters. This function would have a value of unity for \( \lambda_i \in \left[ \frac{\pi}{16}, \frac{\pi}{2} \right] \) and zero for the other frequencies.

In summary, the frequency domain filtering procedure involves the following steps:

1) Compute the discrete Fourier transform of \( \tilde{x}_i \) as \( w_i(\lambda_i) = \frac{1}{\sqrt{n}} \sum_{t=1}^{n} x_t e^{-i\lambda_i t} \) for \( \lambda_i \neq 0 \).

2) Run a cross-frequency regression of \( w_i(\lambda_i) \) on \( \frac{1}{\sqrt{n}} \frac{e^{i\lambda_i t}}{1 - e^{i\lambda_i}} \) for \( \lambda_i \in (0, \pi) \). The resulting regression coefficient is a consistent estimate of the random term \( \tilde{x}_i - \tilde{x}_i \).

3) Apply the indicator function to the residuals from the frequency domain regression and multiply by \( 1 - e^{i\lambda_i} \) to obtain \( \hat{w}_i(\lambda_i) \).

4) Take the inverse Fourier transform \( \hat{x}_i = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \hat{w}_i(\lambda_i) e^{i\lambda_i t} \) to recover the estimated business cycles in the time domain.

References


