

Special Feature C

A Brief Survey Of Density Forecasting In Macroeconomics

by Anthony Tay¹

Introduction

A density forecast of an economic variable is an estimate of the conditional probability density function (p.d.f.) of the possible future values of that variable. For example, a density forecaster might say something like “based on current information, GDP growth over the next year is expected to be normally distributed with mean 3% and standard deviation 0.5%”. A density forecast therefore provides a complete probabilistic description of the possible future realisations of a variable, given some information set. It is a generalisation of the more common point forecast (“GDP growth over the next year is expected to be 3%”) and interval forecast (“GDP growth over the next year is expected to be between 4% and 5%”).

This Special Feature presents a brief survey of density forecasting in macroeconomics. It describes density forecasts, and how they can be constructed, presented, evaluated and used. We will begin with a discussion of some of the disadvantages of point and interval forecasts, and some of the benefits of density forecasts. We look at some examples of density forecasts that are already in regular use, and discuss how such forecasts can be evaluated. Finally, we highlight some of the difficulties and challenges of density forecasting.²

Difficulties With Point And Interval Forecasts

One difficulty with point forecasts is how to interpret them. Should a forecast user interpret a point forecast of, say, 3%, as a mean (so that 3% is the average of all possible outcomes, weighted by their relative probabilities of occurrence) or a median (so that the realisation is as likely to be above 3% as below)? Although the mean and median are equal for symmetric distributions, they are not the same for asymmetric distributions. Another difficulty with point forecasts is that they do not convey forecast uncertainty. There will be times when forecasting is inherently more difficult, and one should expect to make larger forecast errors in volatile episodes than when nothing unusual is happening. At such times, a forecast user would find an indication of forecast

uncertainty useful, and she would put less weight on an uncertain forecast when making decisions or might choose to hedge her subsequent actions more aggressively. The desire to convey forecast uncertainty is one reason why many forecasters now present interval forecasts. For example, an interval forecast of 2.75% to 3.25% output growth over the next year is very different from an interval forecast of 1.75% to 4.25% growth, the latter conveying the notion that there is much more uncertainty about economic conditions.

However, interval forecasts also suffer from problems of interpretability. Does an interval forecast indicate a 0.90 probability, or an interquartile range? Or is the forecaster sure that

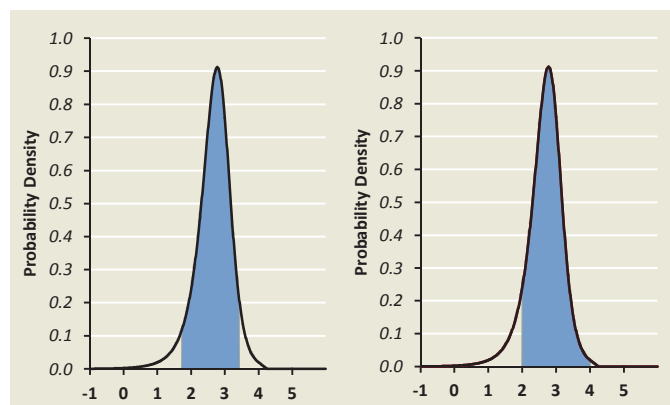
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² See Tay and Wallis (2000) for an early survey of density forecasting in macroeconomics and finance.

the realisation of the variable will fall into that interval? Even if an interval forecast represents a 0.90 probability interval, is this interval centred at the mean, or the median? Is the interval chosen such that the probabilities outside the interval on either side are balanced, or so as to make the 0.90

probability interval the smallest possible such interval? Chart 1 shows two very different interval forecasts with the same probability coverage. Although interval forecasts are more common now, they are almost never accompanied by a full description of what the interval actually says.

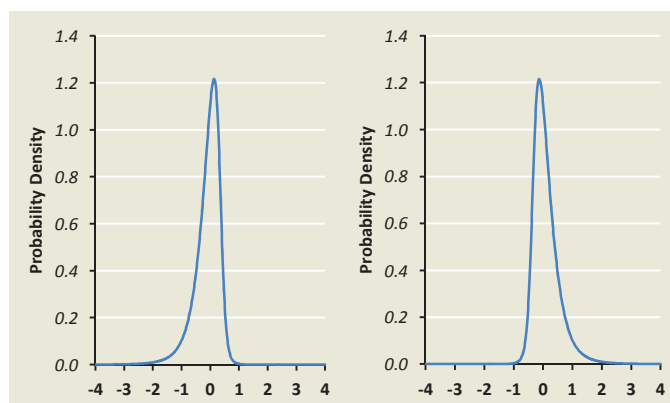
Chart 1
Different Constructions of Interval Forecasts



Because it takes the form of a complete probability density function, a density forecast conveys fully the uncertainty surrounding the forecasts. This is especially useful in situations where there are considerable “downside” risks, relative to “upside” risks (or vice versa). Chart 2 shows two density forecasts, both with zero mode. The density forecast on the left suggests a significant probability of a big negative outcome,

whereas the density forecast on the right shows a considerably more optimistic scenario. Two forecasters with opposing views, one represented by the density forecast on the left, and the other by the density forecast on the right, will give the same modal point forecast. Furthermore, any interval forecast centred at the modes would cover the same probability in either case.

Chart 2
Density Forecasts with Downside and Upside Risks



Another advantage of density forecasts is that it allows users to derive probability forecasts of a given event. This is useful as many of the questions macroeconomists are interested in are most naturally framed in terms of the probability of a specific event occurring, for example, what is

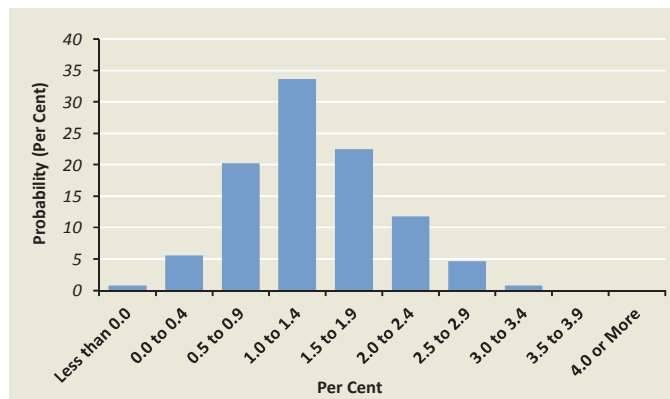
the probability of deflation over the next year? What is the probability that inflation will not fall within a certain target band? Finally, should a user prefer to provide point or interval forecasts, these can be derived directly from the probability density forecast.

Density Forecasting In Practice

In 1968, the Business and Economics Statistics Section of the American Statistical Association (ASA) and the National Bureau of Economic Research (NBER) started the *ASA-NBER Survey*, a quarterly survey of professional macroeconomic forecasters in the US. The Federal Reserve Bank of Philadelphia later assumed responsibility for the survey, which was then renamed as the *Survey of Professional Forecasters*. This survey mostly asks forecasters for their point forecasts

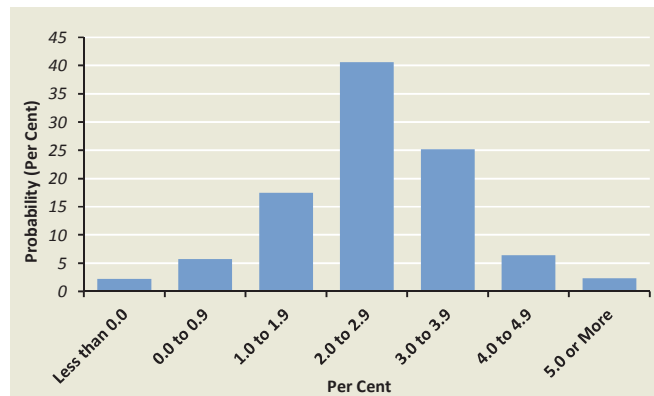
for a range of variables and horizons, but it also asks for probability density forecasts for inflation and output growth in the form of histograms. The forecasters are given a set of intervals, or bins, and asked to assign probabilities to each bin. These are then averaged over all respondents, as shown in Chart 3. In Singapore, density forecasts of GDP growth have been published in the *MAS Survey of Professional Forecasters* since 2001. (Chart 4)

Chart 3
Q1 2015 Density Forecast for US Inflation in 2015



Note: US inflation is measured by the GDP price deflator.

Chart 4
Q1 2015 Density Forecast for Singapore GDP Growth in 2015

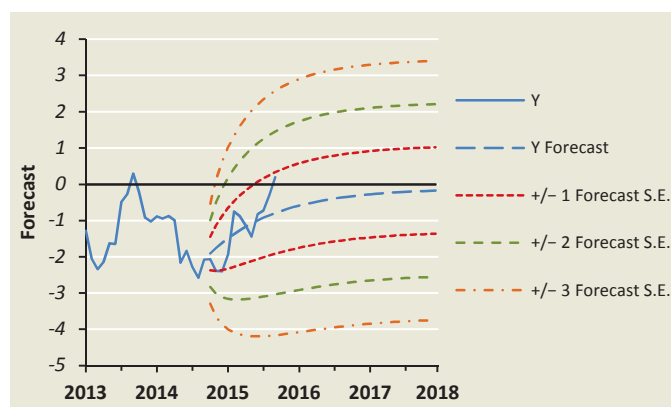


Other prominent examples of density forecasts in macroeconomics include the quarterly density forecasts of one-year ahead UK RPIX inflation³ issued by the Bank of England from Q1 1993 to Q2 2004, and since Q3 1992, by the National Institute of Economic and Social Research (NIESR). The former is based on a ‘two-piece normal distribution’ to allow for asymmetry in the density forecasts (see Wallis, 2004), whereas the NIESR

forecasts are assumed to be normal and centred on the point forecasts generated by a large-scale macroeconomic model. The NIESR forecasts are presented in the form of histograms, whereas the Bank of England forecasts are presented in the form of ‘fan charts’ i.e., overlaid interval forecasts with a range of probability coverage (see Chart 5 for an example of a fan chart).

³ RPIX refers to the Retail Price Index excluding mortgage interest payments.

Chart 5
Example of a Fan Chart



Note: S.E. represents standard error.

Evaluating Density Forecasts

To evaluate point forecasts, we would compare the forecasts with the eventual realisations of the variable forecasted. But how would we compare density forecasts $p_{t|t-1}(\cdot)$ (which are p.d.f.s) with the eventual realisations y_t (which are numbers)? The basic device is to use the quantities:

$$z_t = \text{Prob}(Y_t \leq y_t)$$

implied by the density forecasts. If the density forecasts of a variable Y_t made using information up to time $t-1$ are correct, then the corresponding z_t series should be independently and identically distributed (i.i.d.) uniformly over the interval $[0,1]$. The quantity z_t is often called the probability integral transform (p.i.t.) of the density forecast because it is calculated by taking the integral:

$$z_t = \int_{-\infty}^{y_t} p_{t|t-1}(u) du$$

However, it is nothing more than the cumulative distribution function (c.d.f.) corresponding to the density forecasts, evaluated at the realisations.

There are two parts to this evaluation idea, the ‘uniformity’ part and the ‘i.i.d.’ part. Perhaps the best way to understand the uniformity aspect is to consider probability interval forecasts derived from the density forecasts. A series of 0.90 probability interval forecasts should be correct 90% of the time in the sense that nine out

of ten realisations of the variable should fall into the interval forecasts. The uniformity property says that this should be true of all possible interval forecasts that can be derived from the density forecasts. As another example, think of a forecaster who consistently “gets it wrong”. Suppose we have a “doom-and-gloom” density forecaster who always thinks the economy is going to do worse than it actually does. Such a forecaster consistently puts too much probability on very bad outcomes, so when we calculate the $z_t = \text{Prob}(Y_t \leq y_t)$ series for this forecaster, the values of z_t will tend to be closer to one than they ought to be, i.e., they will be biased upwards. For a good density forecaster, z_t should be evenly distributed from zero to one.

The other aspect of the evaluation principle is the i.i.d. part. Again think of a sequence of 0.90 probability interval forecasts. Not only should these intervals be correctly sized, but we should not be able to use the past sequence of “hits” and “misses” to predict whether the forecaster will be “right” or “wrong” in the future. Thus, the i.i.d. property says that we cannot use past z_t to predict future ones. This is crucial for evaluating density forecasts of economic variables, since dynamics are prevalent in time series of economic variables. A density forecasting model that produces i.i.d. and uniformly distributed p.i.t.’s would be one that describes the dynamic patterns in the data well, while also producing well-calibrated probabilities of events.

This method of evaluating density forecasts was introduced into econometrics by Diebold, Gunther and Tay (1998). In that paper, the authors use informal checks for optimality by plotting histograms and correlograms of the p.i.t. series. Berkowitz (2001) takes a more formal testing approach by further transforming the p.i.t.'s into normal random variables. Under the hypothesis that the density forecasts are correct, z_t has a standard normal distribution, which can be easily tested.

More recently, evaluation procedures have been proposed based on the Kullback-Leibler (KL) 'distance measure', which quantifies how far two probability density functions are from each other. In its original form, the KL measure is not operational since the true density is unknown (it is, after all, the object that we are trying to estimate). The i.i.d. and uniformity result, however, operationalises the KL testing idea, since we can evaluate how far the density of z_t (which we can estimate) is from the uniform distribution.⁴ Details on implementing the KL idea to evaluate and compare density forecasts can be found in Mitchell and Hall (2005) and Bao, Lee and Saltoglu (2007).

There is an interesting problem in density forecast evaluation in that the i.i.d. and uniformity evaluation criteria cannot distinguish between correct density forecasts constructed using different information sets. For instance, suppose Forecaster A produces correct density forecasts of

Y_t based on all past observations of Y_t , while Forecaster B produces correct density forecasts of Y_t using all past observations of Y_t and X_t . Both sets of forecasts will generate i.i.d. and uniformly distributed z_t series as long as the conditional densities are correct relative to their respective information sets. This is implied by the extension of the evaluation idea to multivariate density forecasts in Diebold, Hahn and Tay (1999).

Obviously we would prefer the forecasts based on the larger information set, since we might expect these to be "more accurate", or "more precise". This has led some authors to propose "sharpness" as a criterion. A density forecast is "sharper" if it is more concentrated around a point, an idea which can be measured by the width of, say, the central 0.50 segment of the 0.90 probability intervals. Such an idea is analogous to saying we would prefer density forecasts with smaller variances. This idea is useful in situations where the conditional variances are constant. However, such is not the case with economic data, which often displays time-varying conditional variances.⁵

A simple solution to the problem of the inability of the i.i.d. and uniformity criteria to distinguish between density forecasts using different information sets might be simply to see if the additional information set can predict the z_t series, an idea analogous to what is done in the point forecast literature. However, this idea has not yet been fully explored in the density forecast evaluation literature.

⁴ The KL distance from the density forecast $p_{t|t-1}(y_t)$ to the "true" density $f(y_t)$ is:

$$KL = E_f[\ln f(y_t) - \ln p_{t|t-1}(y_t)]$$

where the expectation is taken with respect to the true density function. It is called a distance because it almost behaves like the usual geometric concept of distance, in that it is never negative, and obeys the triangle inequality (the sum of the shortest distance from point A to B and then to C is never less than the shortest distance from point A directly to point C). It is, however, not a true distance measure because the KL distance from p.d.f. A to p.d.f. B is not the same as the KL distance from p.d.f. B to p.d.f. A.

⁵ The sharpness idea can be found in Gneiting, Balabdaoui and Raftery (2007), although their examples omit time series considerations; see Mitchell and Wallis (2011) for a more in-depth discussion.

Challenges For Density Forecasting In Macroeconomics

There are still rather few examples of density forecasting in macroeconomics, apart from the ones mentioned above. The most prominent examples are those based on surveys that ask for probability estimates in given ranges. Here, much remains to be learnt regarding how such surveys can be improved to give more reliable estimates of forecast uncertainty, and perhaps also a clearer view on the expectations formation process (see Bruine de Bruine *et al.*, 2010).

Another challenge is to develop full macroeconomic density forecasting models. Density forecasts from fully specified macroeconomic models are rare, and tend to make

assumptions such as constant variances and normality of errors, which render the idea of density forecasting somewhat trivial. With better and more abundant data, we can now estimate more interesting models. Autoregressive Conditional Heteroskedasticity (ARCH) models can now be easily constructed on macroeconomic data, leading to less trivial density forecasts by explicitly taking into account the time-varying conditional variances of the underlying data series. The real benefit of density forecasting, however, will probably only be realised with the development of models that allow for time-varying conditional skewness, perhaps driven by data on expectations and sentiments.

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